CANADIAN CONVENTIONS IN FIXED INCOME MARKETS

A Reference Document of Fixed Income Securities Formulas and Practices

Release: 1.2

FORWARD

This document illustrates conventions and formulas currently used by market participants for the calculation of prices, interest payments and yields on securities traded in the Canadian fixed income market. The document has been prepared by a working group of fixed income industry professionals under the auspices of the Investment Industry Association of Canada (formerly the Industry Relations and Representation division of the Investment Dealers Association of Canada (IDA)).

The objective of this document is to provide market participants with a single comprehensive guide to market conventions commonly used in Canada's fixed income market. It will also serve in identifying some of the important ways in which Canadian conventions differ from those of other jurisdictions. In most cases, the document presents existing de facto standards. It is not the intent of this document to introduce new conventions or debate the completeness of those already in practice.

The document consists of two parts. Part I provides a description of the securities together with brief explanations of specific conventions and elements of the valuation formulas. Part II constitutes a quick reference guide that presents the valuation formulas on a case by case basis.

In time we hope this guide facilitates the continued use of uniform practices among Canadian market participants, and further enhances the efficiency and attractiveness of Canada's capital markets.

If you would like to share any comments you may have on the content of this document, please e-mail them to capitalmarkets@iiac.ca.

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1) Section 4.5.1 Short First Coupons Calculated Using Actual/365 – revisions to formula, example added.
2) Section 4.5.2 Long First Coupons Calculated Using Actual/365 – revisions to formula, example added.
3) Section 4.5.3 Short Last Coupons Calculated Using Actual/365 – this is a newly added section
4) Section 6.1 Settlement Accrued Interest Using Actual/365 (Regular Coupon Periods) – revisions to formula

1. INSTRUMENTS

This document examines the following security types:

- Fixed coupon bonds with semi-annual-, quarterly-, and monthly-pay frequencies
- Amortizing securities
- Money market discount notes
- Inflation-linked real return bonds.

1.1 Semi-Annual Pay Bonds

Semi-annual pay bonds form the overwhelming standard for borrowing in the Canadian capital market. Bonds generally pay a fixed annual coupon rate of interest in two equal semi-annual payments. Most bonds have a fixed maturity date.

1.2 Bonds with Other Payment Frequencies

The main types of securities that have been issued in Canada with monthly or quarterly payment frequencies are mortgage-backed securities (monthly) and floating-rate notes (typically quarterly), neither of which are covered in this document. Mortgage-backed security (MBS) conventions have been documented elsewhere (see References section), while floating rate notes (FRNs) are covered by International Swap and Derivatives Association (ISDA) rules.

A limited number of monthly- and quarterly-pay fixed rate bonds have been issued in Canada. Formulas for valuing such securities are presented.

1.3 Amortizing Securities

The types of amortizing securities covered in this document have fixed payment and amortization schedules. This category, for example, includes infrastructure bonds.

To date, a range of different conventions have been used for valuing amortizing bonds in Canada. Due to varying practices amongst market participants, this document presents a single formula for valuation that is based on the same conventions embodied in the valuation formula for non-amortizing Canadian bonds.

From an investor's perspective, the cash flow structure of bonds with mandatory sinking funds is identical to the cash flow structure of an amortizing security. Accordingly, the formulas for valuing amortizing bonds are also appropriate for valuing bonds with mandatory sinking funds.

1.4 Money Market Discount Notes

Several types of discount note are traded in Canada. These include Government of Canada treasury bills, bankers' acceptances, bearer deposit notes and commercial paper. All these securities are issued at prices that are discounted from their principal values using a simple interest rate and the actual/365-day-count convention.

1.5 Inflation Linked Bonds

Inflation-linked bonds, known as real return bonds (RRBs), have been issued by the federal government in Canada since 1991. Certain provincial governments and other issuers have also begun issuing inflation linked bonds in more recent years. Most of these bonds issued in Canada are semi-annual-pay securities that do not repay any principal until the final maturity date. These securities differ from other semi-annual bonds, in that their interest and principal cash flows are linked to the Canadian Consumer Price Index (CPI), as published by Statistics Canada. There have been a few inflation-linked bonds issued in Canada that follow other payment schedules (e.g., amortizers), but these are not covered in this document.

2. DAY-COUNT CONVENTIONS

Day-count conventions determine the method in which the days within an interest payment period are counted.

2.1 Day-Count Conventions for Money Market Securities – Actual/365

Money market instruments accrue interest over periods less than one year. In the Canadian market, the fraction of a year over which interest accrues is calculated using the actual/365-day-count convention, also known as Act/365 day-count basis.

The actual/365-day-count basis considers a year to have 365 days. It does not account for leap years. The fraction of a year represented by any given time period is represented as the actual number of days in the period divided by 365.

Example:

Valuation Date: December 1, 2005
Maturity Date: March 15, 2006

Fraction of a Year = [March 15, 2006 - December 1, 2005]/365
= 104/365
= 0.284931506849315

2.2 Day-Count Conventions for Bonds

Day-count conventions are used when valuing bonds between coupon payment dates. Since consecutive interest payment periods on bonds do not contain equal numbers of days, bond day-count conventions measure the period of time between two dates as a fraction of a coupon period, instead of as a fraction of a year. Two day-count convention are used in the Canadian market.

2.2.1 The Actual/Actual-Day-Count Convention

The actual/actual day-count basis considers the number of days between any two dates to be the actual number of calendar days between the dates[1].

The fraction of a coupon period remaining following a given settlement date is set equal to the actual number of days remaining in the coupon period divided by the actual total number of days in the full coupon period. Similarly, the fraction of a coupon period that has already passed as of a given settlement date is equal to the actual number of days from the last coupon payment date to the settlement date divided by the actual total number of days in the period.

[1] Historically, this day-count basis has been referred to simply as the "actual/actual" basis. However, with the advent of the International Swaps and Derivatives Association (ISDA) version of the actual/actual basis, it has become necessary to distinguish between the two conventions. References to the actual/actual convention in this document refer to the bond market actual/actual convention, the norm for U.S. Treasuries, U.K. Gilts, most European sovereigns and Eurobonds of many currencies, unless otherwise noted. For additional information on differences between bond and swap market conventions, refer to Appendix 1.

Example:

Last Coupon Date: December 1, 2005
Next Coupon Date: June 1, 2006
Valuation Date: March 15, 2006
Total Days in Full Coupon Period (December 1 – June 1) = 182

Fraction of Coupon Period Remaining	=	Actual Days Remaining/Actual Total Days in Period
	=	[June 1, 2006 - March 15, 2006]/182
	=	78/182
	=	0.4285714285714

Fraction of Coupon Period Elapsed	=	Actual Days from Last Payment/Actual Total Days in Period
	=	[March 15, 2006 - December 1, 2005]/182
	=	104/182
	=	0.5714285714286

2.2.2 The Actual/365 (Canadian Bond) Day-Count Convention

The actual/365 (Canadian Bond) day-count convention considers a year to have 365 days, while the length of a coupon period is represented by 365 divided by the number of coupon periods in a year. For the most common Canadian domestic bond structure, which pays a semi-annual coupon, this implies the length of a coupon period is 365/2 = 182.5 days.

Denoting the annual payment frequency (or number of coupon periods per year) as ‘f’, Act/365 (Canadian Bond) measures the fraction of a coupon period represented by a given number of days as follows:

(i) If the number of days of interest accrual is less than the actual number of days in the coupon period:

$$\text{Fraction of Coupon Period} = \frac{Days \cdot f}{365}$$

Which, for semi-annual pay bonds where $f = 2$, reduces to:

$$\text{Fraction of Coupon Period} = \frac{Days}{182.5}$$

(ii) If the number of days of interest accrual exceeds 365/f, or 182.5 days for a semi-annual pay bond:

$$\text{Fraction of Coupon Period} = 1 - \left[\frac{DaysRemainingInPeriod \cdot f}{365}\right]$$

Where

$DaysRemainingInPeriod$ is the actual number of days from Valuation Date to Next Coupon Date

Example of case (i): *Number of days of interest accrual is less than the actual number of days in the coupon period:*

Last Coupon Date:	August 1, 2006
Next Coupon Date:	February 1, 2007
Valuation Date:	September 15, 2006

Fraction of a Coupon Period = [September 15, 2006 - August 1, 2006] • 2/365
= 45 • 2/365
= 0.246575342

Example of case (ii): *The number of days of interest accrual exceeds 365/f:*

Last Coupon Date:	August 1, 2006
Next Coupon Date:	February 1, 2007
Valuation Date:	January 31, 2007

Fraction of a Coupon Period = 1 - [February 1, 2007 - January 31, 2007] • 2/365
= 1 - 2/365
= 0.994520548

3. GENERAL CONSIDERATIONS

3.1 Comparison with U.S. Market Conventions

***Government Bonds**:* The market convention for calculating the price excluding accrued interest on Canadian government bonds is identical to the formula used for pricing U.S. Treasury bonds[2]. This convention extends to bonds with short or long first coupons and to bonds with short last coupons. However, Canadian market conventions differ from U.S. Treasury market conventions in two ways:

1. For trade settlement purposes, accrued interest is calculated using the Act/365 (Canadian Bond) formula, and

2. The amount of interest payable to investors for short or long coupon periods is calculated using the Act/365 (Canadian Bond) convention.

***Corporate Bonds**:* The conventions for corporate bonds in Canada generally follow those for Canadian government bonds, although in isolated cases the interest payable for odd coupon periods may be calculated using some convention other than Act/365 (Canadian Bond). This is in contrast to the U.S., where corporate bonds follow a different convention from their federal counterparts. The bond valuation formula for U.S. corporate bonds uses the US30/360-day-count convention[3].

Money Market: Yields on money market securities in Canada are calculated assuming simple interest and a 365-day year. In contrast, yields on money market securities in the U.S. assume a 360-day year, and are discount yields, rather than yield to maturity.

Real Return Bonds: The structure and inflation-compensation mechanism of real return bonds (RRBs) and their U.S. counterparts, Treasury Inflation-Protected Securities (TIPS), are very similar, notwithstanding that one is linked to the Canadian CPI and the other to U.S. CPI.

The main difference in the design of the two securities is that TIPS have a floor on the inflation-compensation mechanism, such that in the event of deflation, the principal cannot decline below the original issue amount. The different government accrued-interest conventions already noted for nominal bonds apply to RRBs and TIPS as well. Settlement accrued interest on RRBs is calculated using the actual/365 (Canadian Bond) day-count convention.

3.2 Payment Frequency versus Compounding Frequency

There are financial contracts on which the compounding frequency of the yield to maturity is different from the payment frequency of the annual coupon rate. For example, mortgage-backed securities typically pay interest monthly, while yields are expressed on a semi-annual compound basis, for ease of comparison with other domestic bonds. However, all of

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[2] In fact, for many years, Canadian traders used the Monroe calculator to price bonds.

[3] The U.S. 30/360-day-count convention assumes that each month has 30 days and that a year has 360 days.

the securities covered in this document have compounding frequencies that are equal to their annual coupon payment frequencies. Therefore, the formulas in this document use a single mnemonic, “f”, to represent frequency, on the assumption that the compounding frequency of the yield to maturity equals the payment frequency of coupon payments[4].

3.3 Days to Include When Accruing Interest

As a general principle, interest payments on fixed income securities are made in arrears. Thus, the cash payment to an investor occurs on the day following the end of the coupon accrual period. Accordingly, when counting the number of days for the purpose of calculating interest accruals, it is customary to include the first day of the period and to exclude the last day of the period. This approach to counting days is also consistent with the secondary market practice of setting settlement accrued interest equal to the “opening accrued” for the settlement date. The purchaser therefore pays for all interest accrued up to the last day preceding settlement, and is entitled to interest accruing on the settlement date.

3.4 Precision, Truncation and Rounding Conventions

Conventions on truncation, or the use of decimal places, vary depending on the type of instrument being examined and type of calculation being performed. Refer to Appendix 2 for a summary of the existing Canadian practices.

[4] One obvious case where this does not apply is in the formula for money-market equivalent yield. However, in this case it is not necessary to specify a frequency for the yield.

4. CALCULATING PAYMENT AMOUNTS FOR ODD COUPON PERIODS USING ACTUAL/ACTUAL

Bonds are often issued with irregular first or last coupon payment periods, that is, coupon periods that are shorter or longer than the norm. In such cases, the last coupon period would generally be a short period, while an irregular first coupon period may be shorter or longer than a regular coupon period. When calculating the price of a bond, cash flows for such irregular coupon periods are calculated following the actual/actual convention.

4.1 Quasi-Coupon Periods

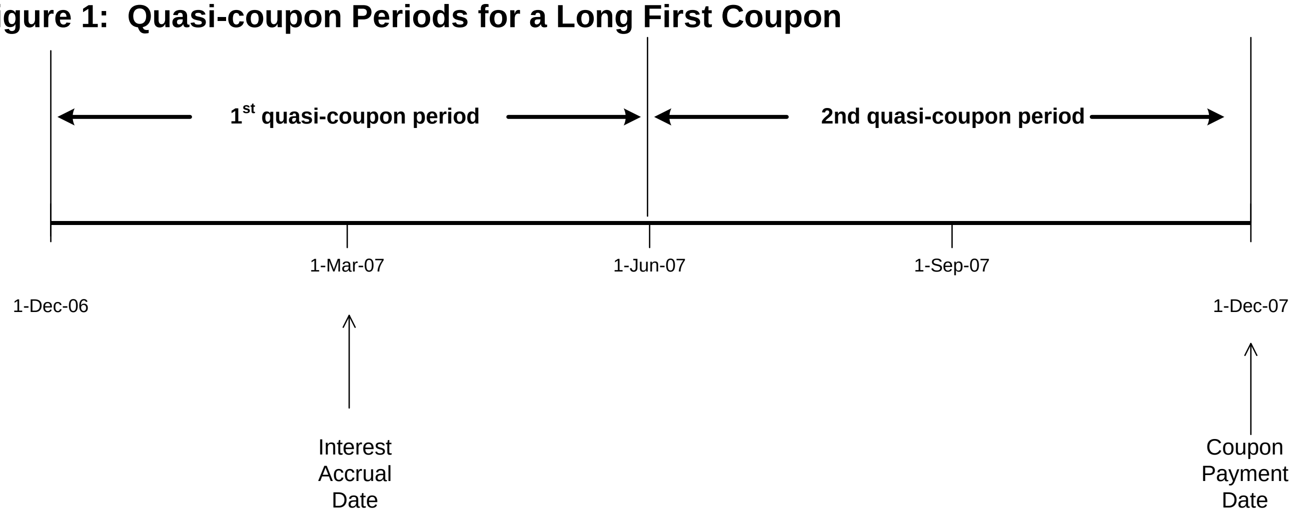
The calculation of payment amounts for irregular coupon periods makes use of the concept of a “quasi-coupon period,” defined as follows:

- For short first coupons, a quasi-coupon period is a hypothetical full coupon period ending on the first coupon date.

- For short last coupons, the quasi-coupon period is a hypothetical full coupon period starting on the penultimate coupon payment date, that is, starting on the next-to-last coupon date.

- A long first coupon period that spans a full coupon period plus a partial period will be divided into two quasi-coupon periods. One is a full coupon period ending on the coupon payment date, while the second is a full coupon period ending on the start date for the later quasi-coupon period. Figure 1 below illustrates this case for a bond with an interest accrual date of March 1, 2007 and a first payment date of December 1, 2007, such that the coupon period is nine (9) months. The interest accrual date is the calendar date from which the security begins to accrue interest.

Figure 1: Quasi-coupon Periods for a Long First Coupon



4.2 Short First Coupons

Using the actual/actual methodology and as illustrated in figure 2, a short first coupon is calculated as follows[5]:

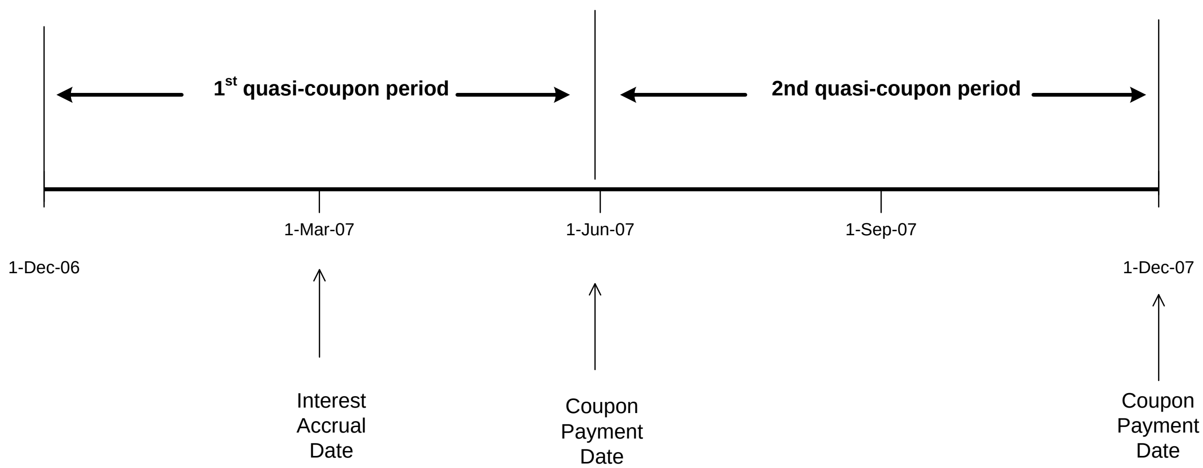
$$C_1 = 100 \cdot \frac{CPN}{f} \cdot \frac{DIC}{DQ}$$

Where:

C_1	=	first coupon payment
CPN	=	annual coupon rate expressed as a decimal
f	=	annual coupon payment frequency
DIC	=	days from interest accrual date to first payment date
DQ	=	days in the quasi-coupon period

See Figure 2.

Figure 2: Quasi-coupon Periods for a Short First Coupon



4.3 Short Last Coupon

Short last coupons occur on securities for which the regular coupon payment schedule does not coincide with the maturity date. For example, a semi-annual pay bond maturing on December 1 might pay coupons each year on October 31 and April 30. The final coupon period would span the October 31 to December 1 period, i.e., one month.

$$C_M = 100 \cdot \frac{CPN}{f} \cdot \frac{DCM}{DQ}$$

[5] A full set of mnemonics is presented in the Appendix 3.

Where:

C_M	=	coupon payment at maturity
CPN	=	annual coupon rate expressed as a decimal
f	=	annual coupon payment frequency
DCM	=	days from penultimate coupon date to maturity date
DQ	=	days in the quasi-coupon period

Example: Calculating the Short Last Coupon on a 5% Semi-annual Bond Using the Actual/Actual Method.

Maturity Date: December 1, 2007
Coupon Dates: April 30 and October 31
Last Coupon Date: October 31, 2007

Days in quasi-coupon period = April 30, 2008 – October 31, 2007 = 182 days
Days of accrual = December 1 – October 31 = 31 days

Coupon payment = 100 • (.05/2) • (31/182) = 0.425824

4.4 Long First Coupon

The formula for the long first coupon is written in a generic fashion to accommodate long coupon periods spanning one or more regular coupon periods, plus a stub period. See Figure 1.

$$C_1 = 100 \cdot \frac{CPN}{f} \cdot \sum_{k=1}^{NQ} \frac{D_k}{DQ_k}$$

Where:

C_1	=	first coupon payment
CPN	=	annual coupon rate expressed as a decimal
f	=	annual coupon payment frequency
NQ	=	number of quasi-coupon periods
D_k	=	days of accrual in quasi-coupon period k
DQ_k	=	total days in quasi-coupon period k

4.5 Calculating Payment Amounts for Odd Coupon Periods Using Actual/365

In Canada, the current convention is to calculate interest payable for odd coupon periods using the Actual/365 (Canadian Bond) method, despite the fact that price/yield calculations assume the first coupon is calculated using actual/actual. This section presents formulas that may be used for this purpose.

4.5.1 Short First Coupons Calculated Using Actual/365

If DIC < 365/frequency, then

$$C_1 = 100 \cdot CPN \cdot \frac{DIC}{365}$$

Where:

C_1	=	first coupon payment
DIC	=	days from interest accrual date to first payment date
CPN	=	annual coupon rate expressed as a decimal

Otherwise (i.e., DIC > 365/frequency):

$$C_1 = 100 \cdot CPN \cdot \left[\frac{1}{f} - \frac{(DQ - DIC)}{365} \right]$$

Where:

f	=	annual payment frequency
DQ	=	days in the quasi-coupon period ending on the first coupon date

In the above formula, the coupon amount is set equal to a full periodic coupon amount minus a straight line accrual amount commensurate with the number of days remaining in the period.

Example: Calculating the Coupon Payable on a 5% Semi-annual Bond with a 183 Day Short First Coupon Period

Issue Date: July 16, 2007
First Coupon Date: January 15, 2008
Maturity Date: July 15, 2020

Days in quasi-coupon period = January 15, 2008 – July 15, 2007 = 184
DIC = January 15, 2008 – July 16, 2007 = 183

$$C_1 = 100 \cdot CPN \cdot \left[\frac{1}{f} - \frac{DQ - DIC}{365} \right]$$

$$= 100 \cdot 5.00\% \cdot \left[\frac{1}{2} - \frac{184 - 183}{365} \right] = 2.486301370$$

4.5.2 Long First Coupons Calculated Using Actual/365

When calculating interest payable for a long interest period, the convention is to use the Act/365 (Canadian Bond) convention for each quasi coupon period within the long coupon period as follows:

If days from interest accrual date to the first quasi-coupon date (DIC*) < 365/frequency, then

$$C_1 = 100 \cdot CPN \cdot \left[\frac{DIC^*}{365} + \frac{NQ-1}{f} \right]$$

Where:

C_1	=	first coupon payment
DIC*	=	days from interest accrual date to first quasi-coupon date
CPN	=	annual coupon rate expressed as a decimal
DQ	=	days in the first quasi-coupon period
f	=	payment frequency
NQ	=	number of quasi-coupon periods

Otherwise (i.e. DIC* > 365/frequency):

$$C_1 = 100 \cdot CPN \cdot \left[\frac{1}{f} - \frac{\left(DQ - DIC^*\right)}{365} + \frac{NQ-1}{f} \right]$$

Example: Calculating the Coupon Payable on a 5% Semi-annual Bond with a Long First Coupon Period

Issue Date: July 16, 2007
First Coupon Date: July 15, 2008
Maturity Date: July 15, 2020

First Quasi-coupon date: January 15, 2008
Days in quasi-coupon period = January 15, 2008 – July 15, 2007 = 184
DIC = January 15, 2008 – July 16, 2007 = 183
NQ = 2

$$\begin{aligned} C_1 &= 100 \cdot CPN \cdot \left[\frac{1}{f} - \frac{DQ - DIC}{365} + \frac{NQ-1}{f} \right] \\ &= 100 \cdot 5.00\% \cdot \left[\frac{1}{2} - \frac{184-183}{365} + \frac{2-1}{2} \right] \\ &= 2.486301370 + 2.50 = 4.986301370 \end{aligned}$$

4.5.3 Short Last Coupons Calculated Using Actual/365

If number of days in last coupon period < 365/frequency:

$$C_M = 100 \cdot CPN \cdot \frac{DCM}{365}$$

Where

C_M	=	coupon payment at maturity
DCM	=	days from penultimate coupon date to maturity date
CPN	=	annual coupon rate expressed as a decimal

Otherwise (i.e. 365/frequency < DCM):

$$C_M = 100 \cdot CPN \cdot \left[\frac{1}{f} - \frac{DQ - DCM}{365} \right]$$

Where

DQ	=	days in the quasi-coupon period starting on the penultimate coupon date.

Example: Calculating the Coupon Payable on a 5% Semi-annual Bond with a Short Last Coupon Period

a) DCM < 365/frequency

Maturity Date: September 1, 2019
Last Coupon Date: July 15, 2019
DCM = September 1, 2019 – July 15, 2019 = 48

$$C_1 = 100 \cdot CPN \cdot \frac{DCM}{365}$$

$$= 100 \cdot 5.00\% \cdot \frac{48}{365} = 0.65734247$$

b) DCM > 365/frequency

Maturity Date: January 14, 2020
Last Coupon Date: July 15, 2019
DCM = January 14, 2020 – July 15, 2019 = 183
DQ = January 15, 2020 – July 15, 2019 = 184
NQ = 2

$$C_1 = 100 \cdot CPN \cdot \left[\frac{1}{f} - \frac{DQ - DCM}{365} + \frac{NQ - 1}{f} \right]$$

$$= 100 \cdot 5.00\% \cdot \left[\frac{1}{2} - \frac{184 - 183}{365} + \frac{2 - 1}{2} \right]$$

$$= 2.486301370 + 2.50 = 4.986301370$$

5. ACTUAL/ACTUAL ACCRUED INTEREST

The convention in Canada is to use actual/actual accrued interest for the purpose of generating a “clean price” from a “dirty price” within the price/yield formula. The clean price is the price quoted in the market, and it excludes accrued interest. The dirty price includes accrued interest and is equal to the present value of the cash flows.

The actual/actual accrued interest methodology is also used for calculating settlement accrued interest on monthly pay bonds. There are four cases to consider:

5.1 Regular Coupon Periods

$$A = 100 \cdot \frac{CPN}{f} \cdot \frac{DCS}{DCC}$$

Where:

A	=	actual/actual accrued interest
CPN	=	annual coupon rate expressed as a decimal
f	=	annual coupon payment frequency
DCS	=	days from last coupon date to settlement date
DCC	=	days from last coupon date to next coupon date

5.2 Short First Coupon Periods

$$A = 100 \cdot \frac{CPN}{f} \cdot \frac{DIS}{DQ}$$

Where:

A	=	actual/actual accrued interest
CPN	=	annual coupon rate expressed as a decimal
f	=	annual coupon payment frequency
DIS	=	days from interest accrual date to settlement date
DQ	=	days in the quasi-coupon period

5.3 Short Last Coupon Periods

$$A = 100 \cdot \frac{CPN}{f} \cdot \frac{DCS}{DQ}$$

Where:

A	=	actual/actual accrued interest
CPN	=	annual coupon rate expressed as a decimal
f	=	annual coupon payment frequency
DCS	=	days from last coupon date to settlement date
DQ	=	days in the quasi-coupon period

5.4 Long First Coupon Periods

The formula for actual/actual accrued for a long first coupon will generally depend on which quasi-coupon period the settlement date falls in. To get around this problem, the symbol DA is used to refer to the number of days over which interest accrues in any given quasi-coupon period. The formula then simplifies to:

$$A = 100 \cdot \frac{CPN}{f} \cdot \sum_{k=1}^{NQ} \frac{DA_k}{DQ_k}$$

Where:

A	=	actual/actual accrued interest
CPN	=	annual coupon rate expressed as a decimal
f	=	annual coupon payment frequency
NQ	=	number of quasi-coupon periods
DA_k	=	days of accrual in quasi-coupon period k
DQ_k	=	total days in quasi-coupon period k

6. SETTLEMENT ACCRUED INTEREST – ACTUAL/365

While actual/actual accrued is used for extracting a clean price from a dirty price, accrued interest for settlement purposes is calculated in Canada using the actual/365 (Canadian Bond) method for quarterly and semi-annual pay securities. However, there is an exception to this convention, for monthly-pay securities. In accordance with IDA (now IIROC) regulation 800.48, accrued interest on monthly-pay securities (that compound interest monthly) should be calculated using the actual/actual method.

There are three cases to consider for actual/365 accrued interest.

6.1 Regular Coupon Periods

If DCS < 365/frequency, then:

$$AI = 100 \cdot CPN \cdot \frac{DCS}{365}$$

Where:

AI	=	actual/365 accrued interest
CPN	=	annual coupon rate expressed as a decimal
f	=	annual coupon payment frequency
DCC	=	days from last coupon date to next coupon date
DCS	=	days from last coupon date to settlement date

Otherwise (i.e., DCS > 365/f) the difference between the full coupon and the accrued interest should be pro-rated over the remaining period.

Examples of Calculating Settlement Accrued for a 5% Semi-annual Pay Bond Maturing February 1, 2008

Example: DCS < 365/f

Settlement Date: December 1, 2005
Last Coupon Date: August 1, 2005

Days of accrued = December 1 - August 1 = 122

Settlement accrued = 100 • 5% • (122/365) = 1.6712

6.2 Short First or Last Coupon Period

The formulas for calculating actual/365 accrued interest for a short first or last coupon are the same as for a regular coupon period.

6.3 Long Coupon Period

There are several possible ways of calculating actual/365 accrued for a long coupon period. One approach is simply to count the total number of days and divide by 365. However, this approach can generate counter-intuitive results in certain cases. A more suitable approach currently being used by market participants is:

1. Separating the long period into one or more full coupon periods plus a stub period (that is, into a series of quasi-coupon periods).

2. Calculating accrued interest for each sub-period in the regular fashion for actual/365 accrued interest.

The formula is:

$$AI = \sum_{k=1}^{NQ} AI_k$$

Where:

AI	=	actual/365 accrued interest
NQ	=	number of quasi-coupon periods
AI_k	=	actual/365 accrued interest for quasi-coupon period k.

7. Discounting for Partial Coupon Periods – Actual/Actual Exponents

When valuing a bond in between coupon payment dates, it is necessary to calculate the present value of the cash flows over the fraction of the current coupon period remaining. For this purpose, the actual/actual methodology is used to calculate the fraction of the current coupon period remaining.

There are four cases to consider.

7.1 Regular Coupon Periods:

$$\alpha = \frac{DSC}{DCC}$$

Where:

α	=	discounting exponent for coupon period in which the settlement date
DSC	=	days from settlement date to next coupon payment date
DCC	=	days from last coupon date to when next coupon date falls

7.2 Short First Coupon Periods:

$$\alpha = \frac{DSC}{DQ}$$

Where:

α	=	discounting exponent for coupon period in which the settlement date falls
DSC	=	days from settlement date to next (first) coupon payment date
DQ	=	days in the quasi-coupon period

7.3 Long First Coupon Periods

$$\alpha = \sum_{k=1}^{NQ} \frac{DR_k}{DQ_k}$$

Where:

α	=	discounting exponent for coupon period in which the settlement date falls
NQ	=	number of quasi-coupon periods
DR_k	=	days remaining in quasi-coupon period k
DQ_k	=	total days in quasi-coupon period k

7.4 Short Last Coupon Periods:

$$\beta = \frac{DCM}{DQ}$$

Where:

β	=	discounting exponent for final coupon period
DCM	=	days from penultimate coupon date to maturity date
DQ	=	days in the final quasi-coupon period

Note that beta defaults to one if the last coupon period is a full coupon period, since DCM will equal DQ in that case.

8. AMORTIZING BONDS

Amortizing bonds in Canada are generally semi-annual pay securities with cash flow structures that include a series of partial principal repayments on interest payment dates prior to maturity. Conceptually, this class of security also includes bonds with mandatory sinking funds, since the cash flow structure of a mandatory sinking fund bond, from an investor's perspective, is identical to that of an amortizing bond.

8.1 Trading Convention

A few of the early amortizing structures issued in the Canadian market were designed to trade on an "original principal balance basis," much like Canadian mortgage-backed securities. Today, however, these securities more commonly trade on a remaining principal balance basis, which is to say that market prices are quoted per $100 of remaining principal. Formulas presented in this section deal with this latter case only.

8.2 Representing Principal Amounts

The valuation of amortizing bonds requires a way of tracking principal outstanding at any given time. ***Therefore, the use of the following definitions is required:***

OPB* = *original principal balance
RPB_t* = *remaining principal balance as at time t

For clarity, on cash flow dates, the remaining principal balance is reduced by the amount of any principal repayment received on that date. Interest for that day will therefore accrue on the lower principal outstanding. This is consistent with counting days for accrued interest, where the end date is excluded.

8.3 Principles Underlying Price-Yield Formula

Reference formulas for calculating prices, yields and accrued interest on amortizing bonds are presented in Section 10. The formula presented is based on the same principles underlying the price-yield formulas for regular bullet bonds in Canada. Specifically:

- The actual/actual (bond) day-count convention is used for the purpose of deriving discount factors when discounting over partial coupon periods.
- Actual/actual (bond) is also used for calculating an accrued interest amount when deriving a clean price from a dirty price and for calculating the amount of an odd coupon within the price-yield formula.
- Act/365 (Canadian Bond) is used for calculating settlement accrued interest and for calculating interest payable for an odd coupon period.

9. REAL RETURN BONDS

Real return bonds (RRBs) are indexed to the Canadian CPI to compensate the holder for any inflation that may have occurred since the bond was issued. Conceptually, Canadian RRBs pay a fixed coupon rate of interest on an indexed principal amount calculated as follows:

$$\text{Indexed Principal}_{Date} = \text{Principal}_{IssueDate} \cdot \frac{CPI_{Date}}{CPI_{IssueDate}}$$

Where:

CPI_{Date} = Consumer price index as at a given date

By indexing the principal, each cash flow on the bond has effectively been indexed. For reference, the original principal can be referred to as the "Real Principal".

9.1 Inflation Indexing Process

In order to value RRBs for any calendar day, a daily CPI series, called the "reference CPI," is defined. The reference CPI for the first day of any calendar month is equal to the CPI for the third preceding calendar month[6]. The reference CPI for any date within a month is calculated by linearly interpolating between the reference CPI applicable to the first day of the month in which such a date falls, and the reference CPI applicable to the first day of the month immediately following. This is an actual/actual day-count interpolation.

$$ref.CPI_{Date} = ref.CPI_M + \frac{t-1}{D}\left[ref.CPI_{M+1} - ref.CPI_M\right]$$

Where:

$ref.CPI_{Date}$	=	reference CPI as at date t
$ref.CPI_M$	=	reference CPI as at the first day of the current month
t	=	calendar day corresponding to the date
D	=	number of days in the month in which the date falls
$ref.CPI_{M+1}$	=	reference CPI as at the first day of the next month

Given the daily Reference CPI series, a daily "index ratio" is defined that reflects all appreciation in the reference CPI occurring since the issue date of the RRB. The index ratio is defined as:

$$Index.Ratio_{Date} = \frac{ref.CPI_{Date}}{ref.CPI_{Base}}$$

[6] For this purpose, the originally published CPI number is used. Revisions to CPI numbers are not used to reprice RRBs.

Where:

$ref.CPI_{Date}$ = reference CPI as at date t

$ref.CPI_{Base}$ = the reference CPI corresponding to the RRB issue date.

9.2 Rounding Convention for the Indexing Process

For the purpose of interpolating reference CPI, calculations are carried to the sixth decimal place and rounded to the nearest five decimal places. Similarly, calculations for the index ratio are carried to six decimal places and rounded to the nearest five decimal places.

Example: Calculating Reference CPI

The example below calculates reference CPI for May 14, 2005:

$\text{Ref CPI}_{\text{May 1, 2005}} = \text{CPI}_{\text{February 2005}} = 125.8$

$\text{Ref CPI}_{\text{June 1, 2005}} = \text{CPI}_{\text{March 2005}} = 126.5$

$$ref.CPI_{May14} = 125.8 + (126.5 - 125.8)\frac{14-1}{31} = 126.09355$$

9.3 Real Price Given Real Yield to Maturity

Aside from the indexing provision, RRBs are identical to conventional semi-annual pay bullet bonds that repay 100 per cent of principal at maturity. Thus, the clean price, given yield to maturity for a RRB, is calculated in a similar fashion. The price resulting from this calculation is known as the “real price.”

9.4 Real Accrued Interest

Real accrued interest as at any given date is the accrued interest calculated on the real principal. This calculation uses the standard actual/365 accrued interest formulas presented in Section 6.

9.5 Settlement Amounts for Real Return Bonds: Nominal Price and Accrued Interest

Settlement amounts for transactions in RRBs are based on the nominal price and nominal accrued interest, which are calculated as follows:

$$\text{Nominal.Price}_{Date} = \text{Real.Price}_{\text{Date}} \cdot \text{Index.Ratio}_{\text{Date}}$$
$$\text{Nominal.Accrued}_{Date} = \text{Real.Accrued}_{\text{Date}} \cdot \text{Index.Ratio}_{\text{Date}}$$

9.6 CPI Re-basing

On occasion, Statistics Canada converts the official time base reference period for the consumer price index (the period for which the value 100 is assigned to the index). This most recently occurred on June 19, 2007 where the base reference period was converted from 1992 to 2002. Changes to the official time base reference period require the Government of Canada via its fiscal agent, the Bank of Canada, to publish, to three decimal places, the conversion factor that is used to rebase historical CPI data. RRB index ratios calculated on and after the rebasing date are based on the new official time base reference period. Historical RRB index ratios calculated prior to this date are not revised.

The Reference CPI_{Base} after the re-basing is a five decimal number that is used in the Index Ratio calculations from the effective date of the CPI re-basing, forward, as shown in the table below.

Table of CPI_{Base} Values

Coupon (%)	**Bond Maturity**	**New Reference CPI_{Base}**	**Previous Reference CPI_{Base}**
4.25	1 December 2021	83.07713	98.86178
4.25	1 December 2026	87.82571	104.51260
4.00	1 December 2031	91.38249	108.74516
3.00	1 December 2036	102.99160	122.56000
2.00	1 December 2041	111.21849	132.35000

Source: www.bankofcanada.ca/en/notices_fmd/2007/not190607.html

PART II

10. REFERENCE FORMULAS

10.1 Semi-annual Pay Bond – Regular Coupon Periods

10.1.1 Price Given Yield to Maturity

$$P = \frac{1}{\left(1+\frac{Y}{2}\right)^{\frac{DSC}{DCC}}} \cdot \left[\frac{100}{\left(1+\frac{Y}{2}\right)^{NC-1}} + \frac{100 \cdot CPN}{2} \cdot \sum_{k=1}^{NC} \frac{1}{\left(1+\frac{Y}{2}\right)^{k-1}} \right] - \frac{100 \cdot CPN}{2} \cdot \frac{DCS}{DCC}$$

Where:

P	=	clean price
Y	=	yield to maturity
DSC	=	number of days from the settlement date to the next coupon date
DCC	=	number of days in the coupon period in which the settlement date falls
NC	=	number of coupon payments remaining
k	=	summation counter
CPN	=	annual coupon rate expressed as a decimal
DCS	=	number of days from the previous coupon date to the settlement date

10.1.2 Settlement Accrued Interest

Less Than 182.5 Days of Accrual

$$AI = 100 \cdot CPN \cdot \frac{DCS}{365}$$

Where:

AI	=	settlement accrued interest
CPN	=	annual coupon rate expressed as a decimal
DCS	=	number of days from the previous coupon date to the settlement date

10.2 Semi-annual Pay Bond – Short First Coupon

10.2.1 Price Given Yield to Maturity

$$P = \frac{1}{\left(1+\frac{Y}{2}\right)^{\frac{DSC}{DQ}}} \cdot \left[100 \cdot \frac{CPN}{2} \cdot \frac{DIC}{DQ} + \frac{100}{\left(1+\frac{Y}{2}\right)^{NC-1}} + \frac{100 \cdot CPN}{2} \cdot \sum_{k=2}^{NC} \frac{1}{\left(1+\frac{Y}{2}\right)^{k-1}} \right] - \frac{100 \cdot CPN}{2} \cdot \frac{DIS}{DQ}$$

Where:

P	=	clean price
Y	=	yield to maturity
DSC	=	number of days from the settlement date to the first coupon date
DQ	=	number of days in the quasi-coupon period ending on the first coupon payment date
CPN	=	annual coupon rate expressed as a decimal
k	=	summation counter
DIC	=	number of days from interest accrual date to first payment date
NC	=	number of coupon payments remaining
DIS	=	number of days from the interest accrual date to the settlement date

10.2.2 Settlement Accrued Interest

$$AI = 100 \cdot CPN \cdot \frac{DIS}{365}$$

Where:

AI	=	settlement accrued interest
CPN	=	annual coupon rate expressed as a decimal
DIS	=	number of days from the interest accrual date to the settlement date

10.3 Semi-annual Pay Bond – Long First Coupon, Settlement in First Quasi-coupon Period

In this example, the bond has a long first coupon period consisting of a full regular coupon period plus a stub period.

10.3.1 Price Given Yield to Maturity

$$P = \frac{1}{\left(1+\frac{Y}{2}\right)^{1+\frac{DSC1}{DQ1}}} \cdot \left[100 \cdot \frac{CPN}{2} \cdot \left(1+\frac{DIC1}{DQ1}\right) + \frac{100}{\left(1+\frac{Y}{2}\right)^{NC-1}} + \frac{100 \cdot CPN}{2} \cdot \sum_{k=2}^{NC} \frac{1}{\left(1+\frac{Y}{2}\right)^{k-1}} \right] - 100 \cdot \frac{CPN}{2} \cdot \frac{DIS}{DQ1}$$

Where:

P	=	clean price
Y	=	yield to maturity
DSC1	=	number of days from settlement date to the first quasi-coupon date
DQ1	=	number of days in the first quasi-coupon period (the stub period)
CPN	=	annual coupon rate expressed as a decimal
k	=	summation counter
DIC1	=	number of days from interest accrual date to first payment date
NC	=	number of coupon payments remaining
DIS	=	number of days from the interest accrual date to the settlement date

10.3.2 Settlement Accrued Interest

$$AI = 100 \cdot CPN \cdot \frac{DIS}{365}$$

Where:

AI	=	settlement accrued interest
CPN	=	annual coupon rate expressed as a decimal
DIS	=	number of days from the interest accrual date to the settlement date

10.4 Semi-annual Pay Bond – Long First Coupon, Settlement in Second Quasi-coupon Period

In this example, the bond has a long first coupon period consisting of a full regular coupon period plus a stub period.

10.4.1 Price Given Yield to Maturity

$$P = \frac{1}{\left(1+\frac{Y}{2}\right)^{\frac{DSC}{DQ2}}} \cdot \left[100 \cdot \frac{CPN}{2} \cdot \left(1+\frac{DIC1}{DQ1}\right) + \frac{100}{\left(1+\frac{Y}{2}\right)^{NC-1}} + \frac{100 \cdot CPN}{2} \cdot \sum_{k=2}^{NC} \frac{1}{\left(1+\frac{Y}{2}\right)^{k-1}} \right] - 100 \cdot \frac{CPN}{2} \cdot \left(\frac{DIC1}{DQ1} + \frac{DC1S}{DQ2}\right)$$

Where:

P	=	clean price
Y	=	yield to maturity
DSC	=	number of days from settlement date to the first coupon payment date
DQ2	=	number of days in the second quasi-coupon period
CPN	=	annual coupon rate expressed as a decimal
DIC1	=	number of days from the interest accrual date to the first quasi-coupon date
DQ1	=	number of days in the first quasi-coupon period (the stub period)
k	=	summation counter
NC	=	number of coupon payments remaining
DC1S	=	number of days from the first quasi-coupon date to the settlement date

10.4.2 Settlement Accrued Interest

$$AI = AI_{Q1} + AI_{Q2}$$

Where:

AI	=	settlement accrued interest
AI_{Q1}	=	settlement accrued interest in the first quasi-coupon period
AI_{Q2}	=	settlement accrued interest in the second quasi-coupon period

Note that the formula for settlement accrued interest in each quasi-coupon period depends on whether there is greater or less than 182.5 days of accrual in the period.

10.5 Semi-annual Pay Bond – Short Last Coupon, Regular First Coupon

10.5.1 Price Given Yield to Maturity

$$P = \frac{1}{\left(1+\frac{Y}{2}\right)^{\frac{DSC}{DCC}}} \cdot \left[\frac{100 \cdot \left(1+\frac{CPN}{2} \cdot \frac{DCM}{DQ_M}\right)}{\left(1+\frac{Y}{2}\right)^{NC-2+\frac{DCM}{DQ_M}}} + \frac{100 \cdot CPN}{2} \cdot \sum_{k=1}^{NC-1} \frac{1}{\left(1+\frac{Y}{2}\right)^{k-1}} \right] - \frac{100 \cdot CPN}{2} \cdot \frac{DCS}{DCC}$$

Where:

P	=	clean price
Y	=	yield to maturity
DSC	=	number of days from the settlement date to the next coupon date
DCC	=	number of days in the coupon period in which the settlement date falls
CPN	=	annual coupon rate expressed as a decimal
DCM	=	number of days from the last coupon date prior to maturity to the maturity date
DQ_M	=	number of days in a full quasi-coupon period beginning on the last coupon date prior to maturity
k	=	summation counter
NC	=	number of coupon payments remaining
DCS	=	number of days from the previous coupon date to the settlement date

10.5.2 Settlement Accrued Interest

Less Than 182.5 Days of Accrual

$$AI = 100 \cdot CPN \cdot \frac{DCS}{365}$$

Greater Than 182.5 Days of Accrual

$$AI = 100 \cdot CPN \cdot \left(\frac{1}{2} - \frac{DCC - DCS}{365} \right)$$

Where:

AI	=	settlement accrued interest
CPN	=	annual coupon rate expressed as a decimal
DCS	=	number of days from the previous coupon date to the settlement date
DCC	=	number of days in the coupon period in which the settlement date falls

10.6 Semi-annual Pay Bond – Short Last Coupon, Short First Coupon

10.6.1 Price Given Yield to Maturity

$$P = \frac{1}{\left(1+\frac{Y}{2}\right)^{\frac{DSC}{DQ}}} \cdot \left[\frac{100 \cdot CPN}{2} \frac{DIC}{DQ} + \frac{100 \cdot \left(1+\frac{CPN}{2} \cdot \frac{DCM}{DQ_M}\right)}{\left(1+\frac{Y}{2}\right)^{NC-2+\frac{DCM}{DQ_M}}} + \frac{100 \cdot CPN}{2} \cdot \sum_{k=2}^{NC-1} \frac{1}{\left(1+\frac{Y}{2}\right)^{k-1}} \right] - \frac{100 \cdot CPN}{2} \cdot \frac{DIS}{DQ}$$

Where:

P	=	clean price
Y	=	yield to maturity
DSC	=	number of days from the settlement date to the first coupon payment date
DQ	=	number of days in the short first quasi-coupon period
CPN	=	annual coupon rate expressed as a decimal
DIC	=	number of days from interest accrual date to first payment date
DCM	=	number of days from the last coupon date prior to maturity to the maturity date
DQ_M	=	number of days in a full quasi-coupon period beginning on the last coupon date prior to maturity
k	=	summation counter
NC	=	number of coupon payments remaining
DIS	=	number of days from the interest accrual date to the settlement date

10.6.2 Settlement Accrued Interest

Less Than 182.5 Days of Accrual

$$AI = 100 \cdot CPN \cdot \frac{DCS}{365}$$

Greater Than 182.5 Days of Accrual

$$AI = 100 \cdot CPN \cdot \left(\frac{1}{2} - \frac{DCC - DCS}{365} \right)$$

Where:

AI	=	settlement accrued interest
CPN	=	annual coupon rate expressed as a decimal
DCS	=	number of days from the previous coupon date to the settlement date
DCC	=	number of days in the coupon period in which the settlement date falls

10.7 Monthly Pay Bond – Regular Coupon Periods

10.7.1 Price Given Yield to Maturity

$$P=\frac{1}{\left(1+\frac{Y}{12}\right)^{\frac{DSC}{DCC}}}\cdot\left[\frac{100}{\left(1+\frac{Y}{12}\right)^{NC-1}}+\frac{100\cdot CPN}{12}\cdot\sum_{k=1}^{NC}\frac{1}{\left(1+\frac{Y}{12}\right)^{k-1}}\right]-\frac{100\cdot CPN}{12}\cdot\frac{DCS}{DCC}$$

Where:

P	=	clean price
Y	=	yield to maturity
DSC	=	number of days from the settlement date to the next coupon date
DCC	=	number of days in the coupon period (month) in which the settlement date falls
NC	=	number of coupon payments remaining
k	=	summation counter
CPN	=	annual coupon rate expressed as a decimal
DCS	=	number of days from the previous coupon date to the settlement date

10.7.2 Settlement Accrued Interest

$$AI = 100 \cdot CPN \cdot \frac{DCS}{365}$$

Where:

AI	=	settlement accrued interest
CPN	=	annual coupon rate expressed as a decimal
DCS	=	number of days from the previous coupon date to the settlement date

10.8 Semi-annual Pay Amortizing Bond – Regular Interest Periods

10.8.1 Price Given Yield to Maturity

$$P = \left(\frac{1}{\left(1+\frac{Y}{2}\right)^{\frac{DSC}{DCC}}} \cdot \left[\sum_{k=1}^{NC} \frac{RP_k + \frac{CPN}{2} \cdot \left(OPB - \sum_{j=1}^{k-1} RP_j \right)}{\left(1+\frac{Y}{2}\right)^{k-1}} \right] - \frac{CPN \cdot RPB_t}{2} \cdot \frac{DCS}{DCC} \right) \cdot \frac{100}{RPB_t}$$

Where:

P	=	clean price
Y	=	annual yield to maturity
DSC	=	number of days from settlement date to first payment date
DCC	=	number of days in the coupon period in which the settlement date falls
NC	=	number of coupon payments remaining
RP_k	=	principal repayment as at cash flow date k
CPN	=	annual coupon rate expressed as a decimal
OPB	=	amount issued (original principal balance)
RPB_t	=	principal balance remaining as at the settlement date
DCS	=	number of days from issue date or last payment date to the settlement date

10.8.2 Settlement Accrued Interest

Less Than 182.5 Days of Accrual

$$AI = \frac{100 \cdot CPN}{RPB_t} \cdot \frac{DCS}{365}$$

Greater Than 182.5 Days of Accrual

$$AI = \frac{100 \cdot CPN}{RPB_t} \cdot \left(\frac{1}{2} - \frac{DCC - DCS}{365} \right)$$

Where:

AI	=	settlement accrued interest
CPN	=	annual coupon rate expressed as a decimal
RPB_t	=	principal balance remaining as at the settlement date
DCC	=	number of days in the coupon period in which the settlement date falls
DCS	=	number of days from the interest accrual date (or previous coupon date) to the settlement date

10.9 Semi-annual Pay Amortizing Bond – Short First Interest Period

10.9.1 Price Given Yield to Maturity

$$P = \left(\frac{1}{\left(1+\frac{Y}{2}\right)^{\frac{DSC}{DQ}}} \cdot \left[\frac{CPN \cdot OPB}{2} \cdot \frac{DIC}{DQ} + RP_1 + \sum_{k=2}^{NC} \frac{RP_k + \frac{CPN}{2} \cdot \left(OPB - \sum_{j=1}^{k-1} RP_j \right)}{\left(1+\frac{Y}{2}\right)^{k-1}} \right] - \frac{CPN \cdot OPB}{2} \cdot \frac{DIS}{DQ} \right) \cdot \frac{100}{OPB}$$

Where:

P	=	clean price
Y	=	annual yield to maturity
DSC	=	number of days from settlement date to first payment date
DQ	=	number of days in the quasi-coupon period ending on the first payment date
CPN	=	annual coupon rate expressed as a decimal
OPB	=	amount issued (original principal balance)
DIC	=	number of days from the interest accrual date to the first payment date
RP_1	=	principal repayment on the first payment date
NC	=	number of coupon payments remaining
k	=	summation counter
RP_k	=	principal repayment as at cash flow date k
DIS	=	number of days from the interest accrual date to the settlement date

10.9.2 Settlement Accrued Interest

Less Than 182.5 Days of Accrual

$$AI = \frac{100 \cdot CPN}{OPB} \cdot \frac{DIS}{365}$$

Greater Than 182.5 Days of Accrual

$$AI = \frac{100 \cdot CPN}{RPB_t} \cdot \left(\frac{1}{2} - \frac{DQ - DIS}{365} \right)$$

Where:

AI	=	settlement accrued interest
CPN	=	annual coupon rate expressed as a decimal
OPB	=	amount issued (original principal balance)
DIS	=	number of days from the interest accrual date to the settlement date
RPB_t	=	principal balance remaining as at the settlement date
DQ	=	number of days in the quasi-coupon period ending on the first payment date

10.10 Money Market Yields

10.10.1 Semi-annual Bonds: One Cash Flow Remaining

The market convention in Canada is to quote a money market equivalent yield on bonds that are in their last coupon period. The formula below converts between price and simple interest yield on a bond with one cash flow remaining.

$$P + AI = \frac{CP + C_M}{1 + YME \cdot \frac{DSM}{365}}$$

Where:

P	=	clean price
AI	=	actual/365 accrued interest
CP	=	100 or call price
C_M	=	coupon payment at maturity
YME	=	money market equivalent yield
DSM	=	days from settlement date to maturity date

10.10.2 Semi Annual Bonds: Two Cash Flows Remaining

Semi-annual-pay bonds in their final year to maturity with two cash flows remaining can be quoted using the usual semi-annually compounded yield to maturity or a money market equivalent yield. In the latter case, the formula below is used:

$$P + AI = \frac{1}{\left(1 + YME \cdot \frac{DSM}{365}\right)} \cdot \left[100 \cdot \frac{CPN}{2} \cdot \left(1 + YME \cdot \frac{DCM}{365}\right) + CP + C_M\right]$$

Where:

P	=	clean price
AI	=	actual/365 accrued interest
YME	=	money market equivalent yield
DSM	=	days from settlement date to maturity date
CPN	=	annual coupon rate expressed as a decimal
DCM	=	days from penultimate coupon date to maturity date
CP	=	100 or call price
C_M	=	coupon payment at maturity

If maturity date falls on a non-business day (weekend or holiday), then the convention in the money market is to roll to the next business day.

10.10.3 Monthly and Quarterly Pay Bonds: Money Market Equivalent Yield

The formula from Section 8.2 is generalized below to address other payment frequencies:

$$P + AI = \frac{1}{1 + YME \cdot \frac{DSM}{365}} \cdot \left[100 \cdot \frac{CPN}{f} \cdot \sum_{k=1}^{NC} \left(1 + YME \cdot \frac{DCM_k}{365} \right) + CP + C_M \right]$$

Where:

P	=	clean price
AI	=	actual/365 accrued interest
YME	=	money market equivalent yield
DSM	=	days from settlement date to maturity date
CPN	=	annual coupon rate expressed as a decimal
f	=	frequency
NC	=	number of coupon payments remaining
DCM_k	=	days from coupon payment k to maturity date
CP	=	100 or call price
C_M	=	coupon payment at maturity

10.10.4 Price and Yield Calculations for Money Market Discount Notes

The price of a typical Canadian domestic money market discount note, such as a treasury bill, commercial paper or bankers' acceptance, is:

$$P = \frac{100}{1 + R \cdot \frac{DSM}{365}}$$

Where:

P	=	clean price
R	=	money market yield to maturity, simple interest
DSM	=	days from settlement date to maturity date

Solving for the yield is done as follows:

$$R = \left[\frac{100 - P}{P}\right] \cdot \left[\frac{365}{DSM}\right]$$

APPENDIX 1: BOND VERSUS SWAP MARKET CONVENTIONS

It is not the purpose of this report to document swap market practices. However, a brief discussion of the key differences may be helpful to understanding bond market conventions, particularly for readers already familiar with swap market practices.

The fundamental difference between bond and swap market conventions concerns the manner in which interest accrues*. For a given nominal annual interest rate, the swap market follows the standard banking practice of accruing interest in equal daily increments throughout each calendar year.* The daily accrual equals the principal of the loan times the nominal annual interest rate divided by the assumed number of days in the year. The interest payable for a given period is then equal to the daily accrual amount times the number of days in the period. Obviously, day-count methodologies are integral to the calculation of interest payable in this approach.

In contrast to swap market practice, bonds pay a periodic rate of interest equal to the nominal annual coupon rate divided by the annual payment frequency. *Thus bonds pay an equal amount of (coupon) interest for each coupon period,* where interest payable equals the periodic interest rate times the bond principal. Several conclusions follow directly from this fact:

1. Abstracting from odd coupon periods, day-counts are irrelevant to the calculation of coupon interest payable on a bond. Day-count conventions are only required to calculate how much of a given coupon has accrued to any date within a coupon period;

2. Since no two consecutive periods contain the same number of days, the “effective” rate of daily interest accrual on a bond changes from coupon period to coupon period (as the number of days in successive periods fluctuates while interest payable remains constant);

3. Day-count conventions such as Act/365 (Canadian Bond) and US30/360, which are commonly used to calculate accrued interest on bonds, necessarily embody ad hoc adjustments to cope with fluctuations in the daily accrual rates from coupon period to coupon period; and

4. The Actual/actual (bond) day-count convention is the most mathematically consistent and appropriate convention for use with bonds because it explicitly accounts for the fluctuations in daily accrual rates from coupon period to coupon period. Not surprisingly, therefore, Actual/actual (bond) is becoming the new global standard for bond valuation.

In contrast to bond market practice, therefore, interest payable on the fixed side of a swap transaction does typically vary with the number of days in each interest period[7]. The example below illustrates the interest amounts payable on a fixed rate bond and the fixed

[7] The ISDA 30/360-day-count convention will generate equal periodic interest payments on a swap when used in conjunction with a certain date roll convention.

leg of an interest rate swap, assuming that the swap is accrued interest using the ISDA Act/365 convention.

Bond versus Swap Cash Flow Conventions

The table below shows the cash flows on a hypothetical two-year bond, with a 5.0% fixed coupon, payable semi-annually, and a hypothetical two-year fixed-for-floating swap. The coupon interest payments on the bond do not depend on the days in each period. They are simply calculated by dividing the annual coupon rate by the payment frequency.

Bond:
Coupon= 5.0%
Payment Frequency = 2, i.e., semi-annual
Coupon Payment Dates: December 1 and June 1
Interest Accrual Date: June 1, 2007

Swap:
Fixed Rate 5.0%
Payment Frequency = 2, i.e., semi-annual
Payment Dates: December 1 and June 1
Business Day Convention: Following
Start Date: June 1, 2007

For a quantity or notional amount of $1 million, the bonds pay a semi-annual coupon of:

$$\frac{0.05}{2} \times \$1,000,000.00 = 25,000.00$$

The cash flows on the swap, in contrast, do vary with the number of days in each period. For example, the first regularly scheduled payment date on the fixed side, December 1 2007, falls on a Saturday, and so the payment date is adjusted to the next business day, Monday, December 3^{rd}, 2007. The first fixed payment is therefore calculated over a 185-day period:

$$0.05 \times \frac{(95+90)}{365} \times \$1,000,00.00 = \$25,342.47$$

This is more than the payment on the bond, because 185 days is more than half of a 365-day year. Subsequent fixed payments are lower, because, in this particular example, each is calculated over 182 days:

$$0.05 \times \frac{(182)}{365} \times \$1,000,000.00 = \$24,931.51$$

Cash Flows on Fixed Leg of Swap and Coupon Payments from a Fixed Rate Bond

Unadjusted Date	*Adjusted Date*	*Days*	*Fixed Swap Cash Flow*	*Bond Coupon Cash Flow*
June 1, 2007	June 1, 2007			
Dec. 1, 2007	Dec. 3, 2007	185	$25,342.47	$25,000.00
June 1, 2008	June 2, 2007	182	$24,931.51	$25,000.00
Dec. 1, 2008	Dec. 1, 2008	182	$24,931.51	$25,000.00
June 1, 2009	June 1, 2009	182	$24,931.51	$25,000.00

The cash flows from the floating leg of the swap are omitted from the preceding table.

Similar differences result when interest on the swap is accrued using the ISDA Actual/actual convention. In the swap market, the Actual/actual convention is similar to the Act/365 convention, with the proviso that the actual number of days in the year in which interest accrues is used in the denominator. ISDA Actual/actual is therefore a means of accounting for leap years when calculating the amount payable over an interest period. Like Act/365, however, under ISDA Actual/actual the daily rate of interest accrual remains constant throughout each year, and interest payable changes from interest period to interest period in accordance with the number of days in each interest period.

In contrast, the bond market version of the actual/actual day-count methodology is employed to calculate how much of a coupon cash flow has accrued to a given date within a coupon period. As discussed above, the coupon payment itself is calculated by taking the nominal annual coupon rate of interest divided by the annual payment frequency. The amount of interest that has accrued to a given date within the coupon period is then calculated as follows:

$$\text{Accrued Interest} = \text{Coupon Payment} \cdot \frac{\text{Actual days elapsed since last coupon}}{\text{Actual number of days in the full coupon period}}$$

APPENDIX 2: ROUNDING AND TRUNCATION PRACTICES

Accuracy of Intermediate Values

Current practice shows that all yield, price and interest calculations should be carried out to the limits of machine precision, and, as a minimum, at least 10 significant decimals.

Dollar Price Accuracy

Bonds
For bonds, dollar prices should be accurate to seven (7) places after the decimal, rounding to six (6) decimals. The rounding is to be applied after solving for the principal (price multiplied by quantity) on the transaction. For calculating principal, full precision is to be used, with the caveat that the resulting principal is rounded to the nearest penny. See the table below.

Money Market
The convention for money market securities is to round prices to three (3) decimals, and to round yields to two (2) decimal places. When calculating principal (price multiplied by quantity) on a trade, the price is rounded to three (3) decimals. This is in contrast to the bond convention where full precision on the price is utilized, with the resultant principal rounded to the nearest penny.

Accrued Interest
Settlement Accrued Interest on a trade is rounded to the nearest penny. However, when calculating present value, full precision is used for accrued interest.

Example:
Bond: Canada 8%, June 1, 2023
Settlement Date: July 9, 2007
Price: 99.987135
Yield: 8.000001
Accrued Interest: $0.83 per $100

Quantity	Principal Quantity • Price	Accrued Interest	Total Settlement Value of Trade
$1,000.00	$999.87	$8.33	$1008.20
$10,000.00	$9,998.71	$83.298	$10,082.00
$100,000.00	$99,987.13	$832.88	$100,820,01
$1,000,000.00	$999,871.35	$8,328.77	$1,008,200.12
$10,000,000.00	$9,998,713.46	$83,287.67	$10,082,001.13
$100,000,000.00	$99,987,134.59	$832,876.71	$100,820,011.30
$1,000,000,000.00	$999,871,345.93	$8,328,767.12	$1,008,200,113.05

APPENDIX 3: MNEMONICS

A	=	actual/actual accrued interest
AI	=	actual/365 accrued interest
AI_k	=	actual/365 accrued interest for quasi-coupon period k
β	=	exponent for short last period
β	=	period discounting exponent for final coupon period same thing?
C_1	=	first coupon payment
C_M	=	coupon payment at maturity
CP	=	100 or the call price
CPN	=	annual coupon rate expressed as a decimal
D_k	=	days of accrual in quasi-coupon period k
DCC	=	days from last coupon date to next coupon date
DCM	=	days from penultimate coupon date to maturity date
DCM_k	=	days from coupon payment k to maturity
DCS	=	days from last coupon date to settlement date
DIC	=	days from interest accrual date to first payment date
DIS	=	days from interest accrual date to settlement date
DQ	=	days in the quasi-coupon period
DQ_k	=	total days in quasi-coupon period k
DR_k	=	days remaining in quasi-coupon period k
DSM	=	days from settlement date to maturity date
DSC	=	days from settlement date to next coupon payment date
f	=	payment frequency
k	=	varying period or date, e.g., quasi-coupon period, cash flow or coupon-payment date
NC	=	number of coupon payments remaining
NQ	=	number of quasi-coupon periods
OPB	=	original principal balance
P	=	clean price
R	=	money market yield to maturity, simple interest
RPB	=	remaining principal balance
RP_k	=	principal repayment as at cash flow date k
α	=	discounting exponent for coupon period in which the settlement date falls

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